Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – statistics and probability

Thursday 21 May 2015 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 13]

Engine oil is sold in cans of two capacities, large and small. The amount, in millilitres, in each can, is normally distributed according to Large $\sim \mathrm{N}(5000, 40)$ and Small $\sim \mathrm{N}(1000, 25)$.

(a) A large can is selected at random. Find the probability that the can contains at least 4995 millilitres of oil. [2]

(b) A large can and a small can are selected at random. Find the probability that the large can contains at least 30 millilitres more than five times the amount contained in the small can. [6]

(c) A large can and five small cans are selected at random. Find the probability that the large can contains at least 30 millilitres less than the total amount contained in the small cans. [5]

2. [Maximum mark: 12]

Eleven students who had under-performed in a philosophy practice examination were given extra tuition before their final examination. The differences between their final examination marks and their practice examination marks were

$$10, -1, 6, 7, -5, -5, 2, -3, 8, 9, -2.$$

Assume that these differences form a random sample from a normal distribution with mean μ and variance σ^2.

(a) Determine unbiased estimates of μ and σ^2. [4]

(b) (i) State suitable hypotheses to test the claim that extra tuition improves examination marks.

(ii) Calculate the p-value of the sample.

(iii) Determine whether or not the above claim is supported at the 5% significance level. [8]

3. [Maximum mark: 9]

A manufacturer of stopwatches employs a large number of people to time the winner of a 100 metre sprint. It is believed that if the true time of the winner is μ seconds, the times recorded are normally distributed with mean μ seconds and standard deviation 0.03 seconds.

The times, in seconds, recorded by six randomly chosen people are

$$9.765\,,\ 9.811\,,\ 9.783\,,\ 9.797\,,\ 9.804\,,\ 9.798\,.$$

(a) Calculate a 99% confidence interval for μ. Give your answer correct to three decimal places. [4]

(b) Interpret the result found in (a). [2]

(c) Find the confidence level of the interval that corresponds to halving the width of the 99% confidence interval. Give your answer as a percentage to the nearest whole number. [3]

4. [Maximum mark: 15]

A random variable X has a population mean μ.

(a) Explain briefly the meaning of

(i) an estimator of μ;

(ii) an unbiased estimator of μ. [3]

(b) A random sample X_1, X_2, X_3 of three independent observations is taken from the distribution of X.

An unbiased estimator of μ, $\mu \neq 0$, is given by $U = \alpha X_1 + \beta X_2 + (\alpha - \beta)X_3$, where $\alpha, \beta \in \mathbb{R}$.

(i) Find the value of α.

(ii) Show that $\mathrm{Var}(U) = \sigma^2\left(2\beta^2 - \beta + \frac{1}{2}\right)$ where $\sigma^2 = \mathrm{Var}(X)$.

(iii) Find the value of β which gives the most efficient estimator of μ of this form.

(iv) Write down an expression for this estimator and determine its variance.

(v) Write down a more efficient estimator of μ than the one found in (iv), justifying your answer. [12]

5. [Maximum mark: 11]

(a) Determine the probability generating function for $X \sim \mathrm{B}(1, p)$. [4]

(b) Explain why the probability generating function for $\mathrm{B}(n, p)$ is a polynomial of degree n. [2]

(c) Two independent random variables X_1 and X_2 are such that $X_1 \sim \mathrm{B}(1, p_1)$ and $X_2 \sim \mathrm{B}(1, p_2)$. Prove that if $X_1 + X_2$ has a binomial distribution then $p_1 = p_2$. [5]